

ASSIGNMENT 7

Question 1:

(a) $\mu(x) = 98.1$, $\sigma = 0.67$

find $p(x < 97.5)$

Solution

$$P(x < 97.5) = P\left(\frac{x - \mu}{\sigma} < \frac{97.5 - 98.1}{0.67}\right)$$

$$= P(z < -0.8955)$$

$$= \underline{\underline{0.1852}} \rightarrow \text{from std. normal tables.}$$

(b) $P(x > 97.5)$

Solution

$$P(x > 97.5) = 1 - P(x < 97.5)$$

$$= 1 - 0.1852$$

$$= \underline{\underline{0.8148}}$$

(c) $P(x < 97.5)$, when $n = 9$

Solution

$$P(\bar{x} < 97.5) = P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{97.5 - \mu}{\sigma/\sqrt{n}}\right) = P\left(z < \frac{97.5 - 98.1}{\frac{0.67}{\sqrt{9}}}\right)$$

$$= P(z < -2.6866) = P(z < -2.6866)$$

$$\approx \underline{\underline{0.0036}} \rightarrow \text{from std. normal tables}$$

Question 2.

(a) solution:

probability that it is purple = $1 - \text{Probability that the flower is yellow}$

$$= 1 - 0.53$$

$$= \underline{\underline{0.47}}$$

(b) solution

using binomial probability distribution,

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= \binom{5}{3} (0.53)^3 (0.47)^2 + \binom{5}{4} (0.53)^4 (0.47)^1 + \binom{5}{5} (0.53)^5 (0.47)^0$$

$$= \underline{\underline{0.5561}}$$

Question 2. (c)

since $n > 30$, we use approximation to normal.

$$\text{mean} = np = 263 \times 0.53 = 139.39.$$

$$\text{std. dev} = \sqrt{npq} = \sqrt{263 \times 0.53 \times 0.47} \approx 8.094.$$

$$P(x \leq 150) = P\left(\frac{x - \mu}{\sigma} < \frac{150 - 139.39}{8.094}\right)$$

$$= P(Z < 1.3108)$$

$$\approx \underline{\underline{0.9050}} \rightarrow \text{From std. normal tables.}$$

Question 3.

(a) 95% confidence interval for the mean

Solution:

$$\text{mean} = \frac{1}{8} (77.7 + 84.8 + \dots + 97.2)$$

$$= 88.21$$

$$\text{std. dev} = \sqrt{\frac{1}{n-1} \sum_{i=1}^8 (x_i - \bar{x})^2} = \frac{1}{7} \left[(77.7 - 88.21)^2 + (84.8 - 88.21)^2 + \dots + (97.2 - 88.21)^2 \right] \\ \approx 8.0928$$

$$\text{Std. Error (S.E)} = \frac{\sigma}{\sqrt{n}} = \frac{8.0928}{\sqrt{8}} \approx 2.8612$$

CI at (7 degrees of freedom) ≈ 2.365

$$\Rightarrow \bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= [88.21 \pm (2.365 \times 2.8612)]$$

$$= \underline{\underline{81.4 < \mu < 94.97}}$$

Question 3. (b)

99% confidence interval for mean.

Solution

$$\bar{x} = 88.21, \quad Z_{\alpha/2} = 3.499 \text{ - from T-table at 7 d.f.}$$

$$s = 8.0928$$

$$S.E = 2.8612$$

Confidence interval

$$= [88.21 \pm 3.499 \times 2.8612]$$

$$= \underline{\underline{78.20 < \mu < 98.22}}$$

Question 3. (c)

95% CI for standard deviation

Solution

$$95\% \text{ CI} = \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$$

$$\text{now } \alpha/2 = \frac{0.05}{2} = 0.025, \quad 1-\alpha/2 = 0.975$$

$$\chi^2_{0.025} = 16.0128 \text{ ---- from Excel using CHINV}(0.025, 7)$$

$$\text{Lower bound} = \sqrt{\frac{7 \times 65.4927}{16.0128}} = 5.3507$$

$$\text{Upper bound} = \chi^2_{1-0.025} = 1.6899 \text{ from excel formula CHINV}(0.975, 7)$$

$$= \sqrt{\frac{7 \times 65.4927}{1.6899}} = \underline{\underline{16.4708}}$$

$$\Rightarrow 95\% \text{ confidence interval will be } := \underline{\underline{5.3507 < \sigma < 16.4708}}$$

Excel ribbon showing tabs: Clipboard, Font, Alignment, Number, Styles, Insert. The Font tab is active, showing Calibri font, size 11, and various formatting options like Bold, Italic, Underline, and Color. The Alignment tab shows options for text alignment and Merge & Center. The Number tab shows currency and percentage symbols. The Styles tab shows Conditional Formatting, Format as Table, and Cell Styles. The Insert tab shows options for inserting tables, charts, and links.

Formula bar: A18, X, ✓, fx, mean

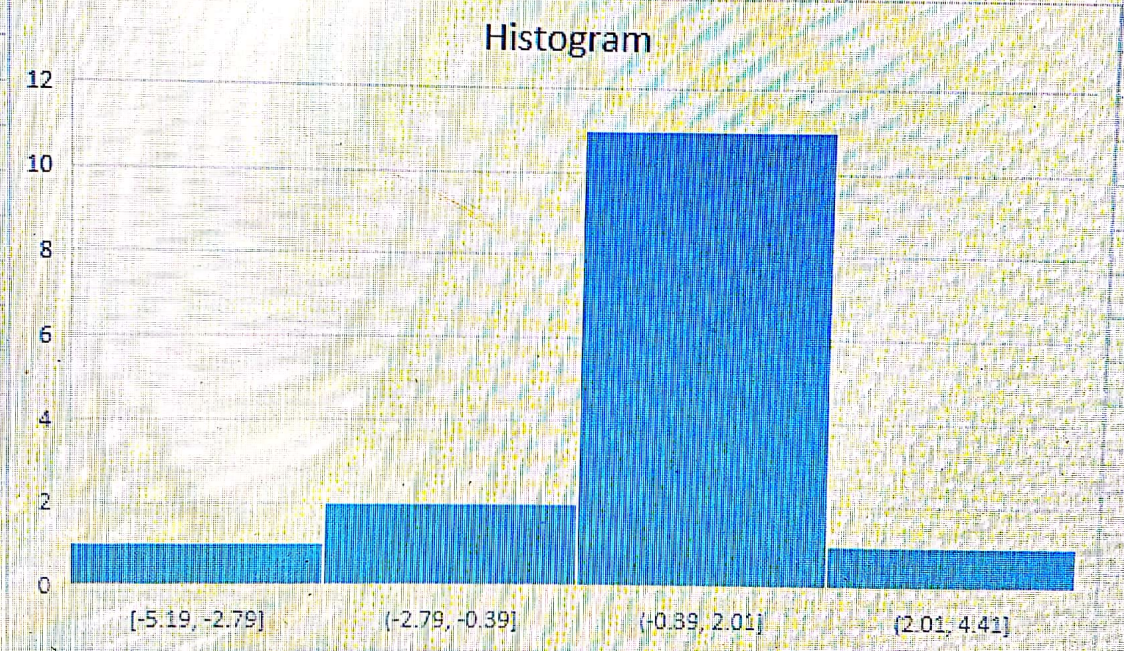
Excel grid headers: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P

	-5.19
	-1.2
	-0.5
	-0.33
	-0.15
	-0.15
	-0.07
	0.02
	0.02
	0.28
	0.37
	0.45
	1.76
	2.8

mean -0.136
std.dev 1.687901

Question 4(a)

Yes, people tend to turn in one direction, i.e. to the left (left skewed) as per the below histogram



Qn 4 (a) Continuation

Test statistic is computed as follows:

$$H_0: \mu_0 = 0$$

$$H_a: \mu_0 \neq 0$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, \quad \bar{x} = -0.136$$
$$s = 1.688$$
$$n = 15$$

$$\Rightarrow t = \frac{-0.136 - 0}{\frac{1.688}{\sqrt{15}}} \approx -0.312$$

Since it is observed that $|t| = 0.312 \leq t_c = 2.145$, we conclude that the mean angle does not differ from zero.

Qn 4 (b)

YES, based on the test, the following statement is justified;
"people do not have tendency to turn more in one direction, on average, than the other direction"